

## MATH 579: Combinatorics Exam 1

Please read the following instructions. For the following exam you may not use any papers, books, or computers. You may use a calculator. Please turn in **exactly four** problems. You must do problems 1-3, and one more chosen from 4-6. Number 7 is optional. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 50 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 20 and 40. This will then be multiplied by  $\frac{5}{2}$  for your exam score.

### Turn in problems 1,2,3:

1. How many four-letter words, drawn from the usual 26 letters, contain exactly three different letters?
2. Calculate  $S(6, 3)$ .
3. How many solutions are there to  $x_1 + x_2 + x_3 = 30$  in nonnegative integers, with  $x_1, x_2, x_3$  distinct?

### Turn in exactly one more problem of your choice:

4. Find a closed formula for  $S(n, n - 2)$ , for all  $n \geq 3$ . Be sure to prove your answer.
5. Let  $p_k(n)$  denote the number of partitions of  $n$  into  $k$  parts, and let  $t \in \mathbb{N}$ . Prove that  $\lim_{n \rightarrow \infty} p_{n-t}(n)$  exists, and find this limit.
6. Count the number of functions  $f : A \rightarrow B$ , with  $|A| = a$ ,  $|B| = b$ , with the elements of  $A$  identical to each other, and the elements of  $B$  identical to each other. Be sure to prove your answer.

### You may also turn in the following (optional):

7. Describe your preferences for your next group assignment. (will be kept confidential)